

Magnetic g -Factors with a Surface Delta Interaction

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Abstract

Using an attractive surface delta interaction we obtain wave functions for 2 protons (or proton holes) in the $f_{5/2}$ and $p_{3/2}$ shell. We take the single particle energies to be degenerate. We obtain the remarkable result that the magnetic g -factors of the lowest 2^+ state is equal to 1 and the same is true for the lowest 4^+ state. Only the orbital part of the g -factors contribute – the spin part cancels out. Then shell model calculations are performed for these same g factors in a larger space and with realistic effective interactions.

1 The Surface Delta Interaction

The surface delta interaction (SDI) of Green and Moszkowski [1] and Arvieu and Moszkowski [2] has proven to be a very useful schematic interaction. It can be used to find the hidden simplicity in complex calculations. This interaction has been extensively discussed in Talmi's book [3].

The matrix element of the SDI interaction can be written as follows:

$$\langle [j_1 j_2] SDI [j_3 j_4] \rangle = C_0 f(j_1, j_2) f(j_3, j_4) \quad (1)$$

where we have

$$f(j_1, j_2) = (-1)^{j_2 + \frac{1}{2}} \sqrt{\frac{2(2j_1 + 1)(2j_2 + 1)}{(2J + 1)(1 + \delta_{j_1 j_2})}} \times \left\langle j_1 j_2 \frac{1}{2} \left(\frac{-1}{2} \right) \middle| J 0 \right\rangle \quad (2)$$

Note that the expression is separable and so, as indicated by Talmi [3] it is easy to obtain the lowest state wave functions (see Eq. 12.49). In our notation we use

$$\psi^J = \sum f(j_1, j_2) [j_1 j_2]^J \quad (3)$$

2 Wave Functions and g -Factors

We consider 2 proton particles or 2 proton holes in either the $f_{5/2}$ or $p_{3/2}$ orbitals. As an orientation the 2 hole system could be ^{86}Kr . We take the single particle energies to be degenerate. We choose C_0 to be negative. We find that the wave function of the $J = 4^+$ state is

$$a[f_{5/2} f_{5/2}]^4 + b[f_{5/2} p_{3/2}]^4 \quad (4)$$

and similarly the wave function of the 2^+ state is

$$c[f_{5/2} f_{5/2}]^2 + d[f_{5/2} p_{3/2}]^2 + e[p_{3/2} p_{3/2}]^2 \quad (5)$$

The following values of the coefficients are obtained by matrix diagonalization:

$$\begin{aligned} J = 4^+ &\rightarrow a = -0.447189, \quad b = 0.894444 \\ J = 2^+ &\rightarrow c = 0.692819, \quad d = -0.48992, \quad e = 0.5291337 \end{aligned}$$

Using the bare g -factors for the protons ($g_l=1$, $g_s=5.586$) the values of $g(J)$ for the 2 hole system are:

$$\begin{aligned} g(4^+) &= 0.3448a^2 + 1.1638b^2 \\ g(2^+) &= 0.3448c^2 + 0.5268d^2 + 2.5287e^2 \end{aligned}$$

With the above values we find the interesting result that $g(2^+)$ is equal to $g(4^+)$ and both are equal to one or more properly stated both are equal to g_l .

To gain further insight we did calculations with $g_l = 0$ but keeping g_s unchanged. The expressions are

$$\begin{aligned} g(4^+) &= -0.798a^2 + 0.1995b^2 \\ g(2^+) &= -0.798c^2 - 0.57633333d^2 + 1.862e^2 \end{aligned}$$

We find that both $g(4^+)$ and $g(2^+)$ vanish when we set $g_l = 0$ i.e. the spin contributions cancel out. If we set $g_s = 0$ i.e. we include only the orbital part and the expressions become

$$\begin{aligned} g(4^+) &= 1.14285714286a^2 + 0.9642857144b^2 \\ g(2^+) &= 1.14285714286c^2 + 1.103188889192d^2 + 0.66666667e^2 \end{aligned}$$

and one sees that again $g(4^+) = g(2^+) = 1$. This behavior is connected with a combination of pseudo-spin symmetry developed by Hecht et al. [4] and Arima et al. [5]. Insights into the origin of pseudospin symmetry via the Dirac equation has been giving by Ginocchio and Leviatan [6]. Also relevant is the quasi-spin formulation of Kerman [7] and Talmi's generalized seniority scheme [3]. From the pseudo-spin formulation one can map the orbits $f_{5/2}$ and $p_{3/2}$ into $d_{5/2}$ and $d_{3/2}$. We further note that the surface delta interaction is a quasi-spin conserving interaction. In the d shell the expressions for the g -factors are a bit more complicated because the magnetic moment operator can connect $d_{5/2}$ and $d_{3/2}$. The expressions are:

$$\begin{aligned} g(4^+) &= 1.9172a^2 + 1.2293b^2 + 0.917199968ab \\ g(2^+) &= 1.9172c^2 + 1.764333333d^2 + 0.0828e^2 + 2.5942332913cd - 1.98137832564de \end{aligned}$$

One can verify that with the surface delta interaction $g(4^+) = g(2^+) = 1$. A simpler argument in the d shell is that with a spin independent interaction (i.e. Wigner interaction) LS coupling holds. Thus the $J = 2^+$ state has $L = 2$, $S = 0$ whilst the $J = 4^+$ state has $L = 4$, $S = 0$. Hence only the orbital angular momentum contributes to the g -factors. However in the $f_{5/2}$ - $p_{3/2}$ space it is far less obvious. We briefly comment on single particle energies. In the theoretical work of Hjørth-Jensen et al. [8] the $f_{5/2}$ - $p_{3/2}$ single particle splitting is 4.4 MeV. However this is in anticipation of a large scale shell model calculation with many valence nucleons relative to ^{40}Ca . If we look more locally we note that for ^{85}Br (one proton less than ^{86}Kr) the ground state has $J = \frac{3}{2}^-$ with the $J = \frac{5}{2}^-$ state at an excitation energy of 0.345 MeV. (This is in stark contrast to the naive using of the single particle energy splitting in [8] which would lead to a $J = \frac{5}{2}^-$ ground state with the $J = \frac{3}{2}^-$ state at 4.4 MeV.) So taking the single particle energies degenerate is not so bad.

It should be pointed out that the experimental g -factor of the 2^+ state is very close to 1. In the work of T.J. Mertzimekis et al. [9] they quote the value 1.12 (14). Recent measurements by Kumbartzki et al. [11] for the 2^+ gives a value consistent with this 1.10(5). They have a new result for the 4^+ 1.07(15).

Amusingly, both $g(2^+)$ and $g(4^+)$ are close to unity, as is given by the surface delta interaction in the small space. This by itself does not mean that the wave functions of this interaction are literally correct but this schematic model gives us ideas of examining the orbital and spin contents of magnetic g factors.

Most g -factors of excited states of even-even nuclei are close to $\frac{Z}{A}$ so that a g -factor of 1 is unusual in any circumstance. Things that conspire to yield this result are that we have a closed shell of neutrons $N = 50$ and the closeness of the $f_{5/2}$ and $p_{3/2}$ single particle energies. We however do not have any explicit empirical evidence that the g -factor is dominantly orbital.

Although in this simple model the expectation value of g_s is calculated to be zero in ^{86}Kr this does not mean that we can assign a quantum number $S = 0$ for it. Expressed in LS coupling in terms of basis states $[(1_1 1_2)^L (1/2 1/2)^S]^J$ the wave function is fairly complicated e.g. for $J = 4^+$.

$$\Psi = \begin{array}{ccc} 0.07376[(33)^3 1]^4 & -0.21187[(33)^4 0]^4 & -0.38684[(33)^5 1]^4 \\ -0.16903[(31)^3 1]^4 & +0.58717[(31)^4 0]^4 & +0.6547[(31)^4 1]^4 \end{array} \quad (6)$$

We thus have admixtures of $L=3, 4$ and 5 and $S=0$ and 1 in this wave function. Note that the basis state $[(33)^4(\frac{1}{2}\frac{1}{2})^1]^4$ is symmetric and therefor must be excluded.

It should also be pointed out that the probability of the (33) configuration is 0.2 and the probability of the (31) is 0.8 . The expectation value S then for the first three terms “ (33) ” is -0.1428 and for the last three terms “ (31) ” is 0.1428 giving a net value of 0 . But clearly L and S are not good quantum numbers. It is not a priori obvious that the above wave function would give such a simple result for the g factors and its purely orbital character.

We next look at the experimental data. The observed energies of the first 2^+ and 4^+ states in ^{86}Kr are 1.565 MeV and 2.350 MeV respectively. However with the SDI they would be degenerate. With $C_0 = -0.5$ MeV they would be at an excitation energy of 1.786 MeV. We now introduces a single particle energy splitting

$$\epsilon^{-1}(f_{5/2}) - \epsilon^{-1}(p_{3/2}) = 0.4\text{MeV} \quad (7)$$

from the ^{87}Rb spectrum (single hole). We are interested in seeing how this affects the g -factors. We find

$$\begin{aligned} J = 4^+ &\rightarrow a = -0.297292, \quad b = 0.954787 \\ J = 2^+ &\rightarrow c = 0.368227, \quad d = -0.390574, \quad e = 0.84378 \end{aligned}$$

which gives us $E(2^+) = 1.684$ MeV and $E(4^+) = 1.824$ MeV. The energy splitting goes in the right direction. The values of the g -factors are $g(2^+) = 1.927$ and $g(4^+) = 1.091$. We see that $g(4^+)$ is still close to 1 but $g(2^+)$ becomes very large. This is because the g -factor of a $p_{3/2}$ proton (2.529) is much larger than that of an $f_{5/2}$ proton (0.3448). Still it might not be unreasonable to say that the SDI calculation is suggestive of what one should look for in more realistic calculations.

Talmi cites in his 1993 book [3] another example, the $g_{7/2}-d_{5/2}$ space where the single particle splittings are also small. He also has a nice discussion (p. 445) of pseudo-orbital angular momentum.

3 Large Space Shell model calculations

In the work of Kumbatrzki et al. [10] large space shell model results were presented for ^{86}Kr energy levels and g factors. The interaction consisted of monopole corrected G matrix elements based on the CDBonn interaction of Sieja et al. [11]. The results for the g factors were close to experimental ones. The theory(experiment) values of the energy levels were 1.613 (1.655) MeV and 2.265 (2.250) MeV for $J=2^+$ and 4^+ respectively. The respective g factors were 1.03 ($1.10(5)$) and 0.99 (1.03 (14))) In these calculations only proton valence particles were active whilst the neutrons formed a closed $N=50$ core. The protons were allowed to be in 4 shells $1f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$ and $0g_{9/2}$. The value of g_s was quenched by a factor of 0.75 .

We now address and expand upon what was not previously considered in [10], how much of the g factor comes from the orbital part of the M1 operator and how much comes from the spin. We do this with the results from both the calculations in [10], [12] but also examine two additional effective interactions in the same model space, JUN45 [13] and jj4b [14]. The results for these interactions and for the wavefunctions are presented in Table 1.

In each case, we find that the g factors are close to 1 and dominated by the orbital contribution.

Table 1: g-factors and wavefunction configurations for 2_1^+ and 4_1^+ in large scale shell model calculations.

Interaction	blat[12]	JUN45	JJ4B	SDI interaction
$g(2_1^+)$ (total,orbital,spin)	1.03, 0.99, 0.04	1.22, 0.93, 0.29	1.09 ,0.97,0.12	1,1,0
$g(4_1^+)$ (total,orbital,spin)	0.99, 1.00, -0.01	1.03, 0.99,0.04	1.03 ,0.99,0.04	1,1,0
Wavefunction of 2_1^+				
3410	0.05	<0.05	0.07	
4202	0.06	<0.05	0.06	
4220	0.08	<0.05	<0.05	
4310	0.13	<0.05	0.07	
4400	0.20	0.06	0.19	0.48
5210	<0.05	0.05	<0.05	
5300	<0.05	0.13	0.08	0.24
6110	0.06	0.11	0.05	
6200	0.23	0.45	0.27	0.28
Wavefunction of 4_1^+				
5210	0.08	<0.05	<0.05	
5300	0.70	0.69	0.63	0.80
5102	<0.05	0.06	<0.05	
4400	<0.05	0.05	0.09	0.20
4202	<0.05	<0.05	0.05	

In looking at the wavefunctions we see that the wave functions are more complicated than those in the small space of sections 2.

However the combined occupations of orbits not included in the small space , namely $p_{1/2}$ and $g_{9/2}$, is not large. We note that for the 4^+ state $b^2=0.799$. in the small space In the large space the weight ranges from 0.63 to 0.70 , not so different.

This suggests that the simple model in the previous section is not an unreasonable starting point. Of course ultimately large space calculations have to be performed to compare with the data.

We also have results for the second 2^+ state where the g factor in the case of the JUN45 interaction continues to be orbital dominated, 1.0596 (0.9873,0.1830). In a vibrational collective model one can obtain such a result—orbital dominance ,where the 2^+ and 4^+ states are part of a 2 phonon triplet.

We thus have 2 types of models which lead to orbital dominance—one a schematic SDI calculation in a small space and a more realistic calculation in a larger space. One important lesson in all this is that one does not have to have pure LS wave functions to get results where almost all the contributions are orbital.

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